Chapter 7 (our shortest chapter!)

(Some material taken from McGraw-Hill/College Physics/Giambattista, the majority from your Katz textbook)

According to **Newton's law of universal gravitation,** any two objects exert gravitational forces on each other that are proportional to the masses (m_1 and m_2) of the two objects and **inversely proportional** to the square of the distance (r) between their centers.

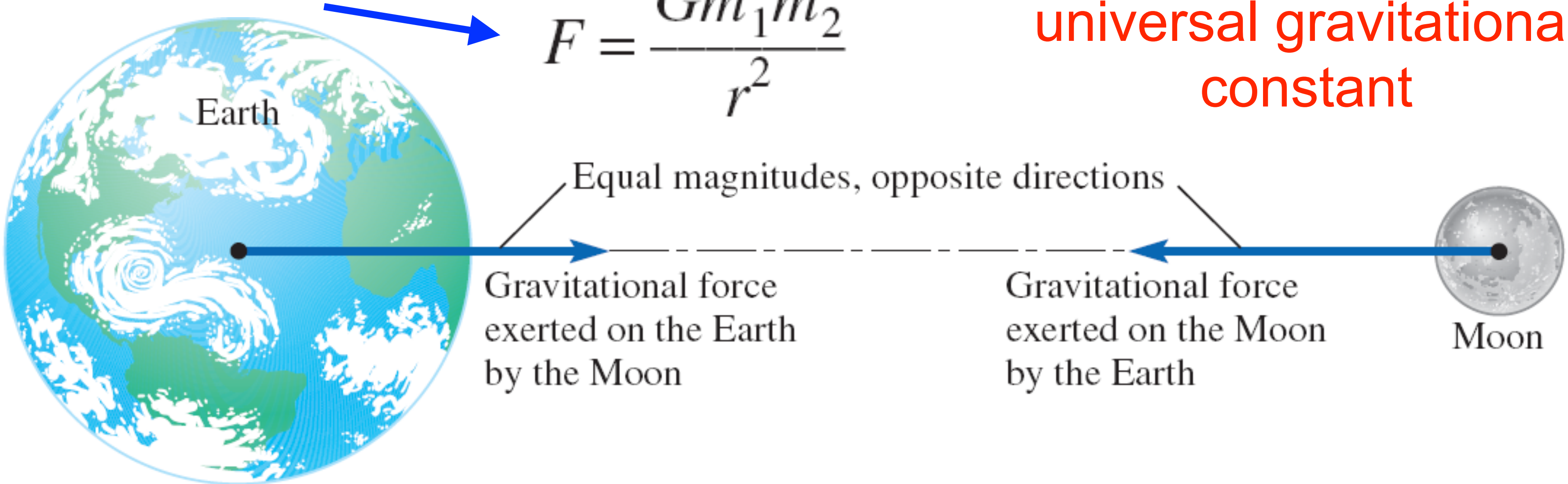
Don't need to remember G

Scalar? Or vector?

$$F = \frac{Gm_1m_2}{r^2}$$

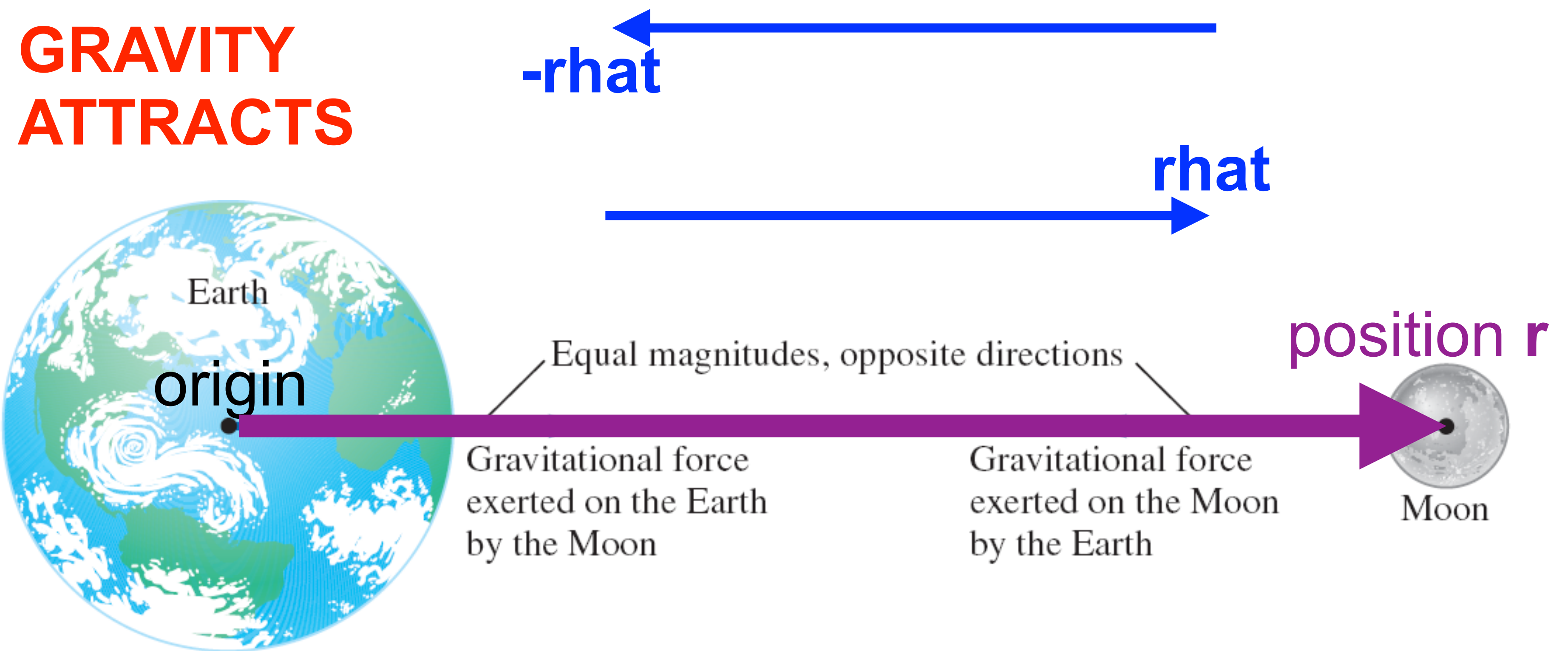
$$G = 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

universal gravitational constant

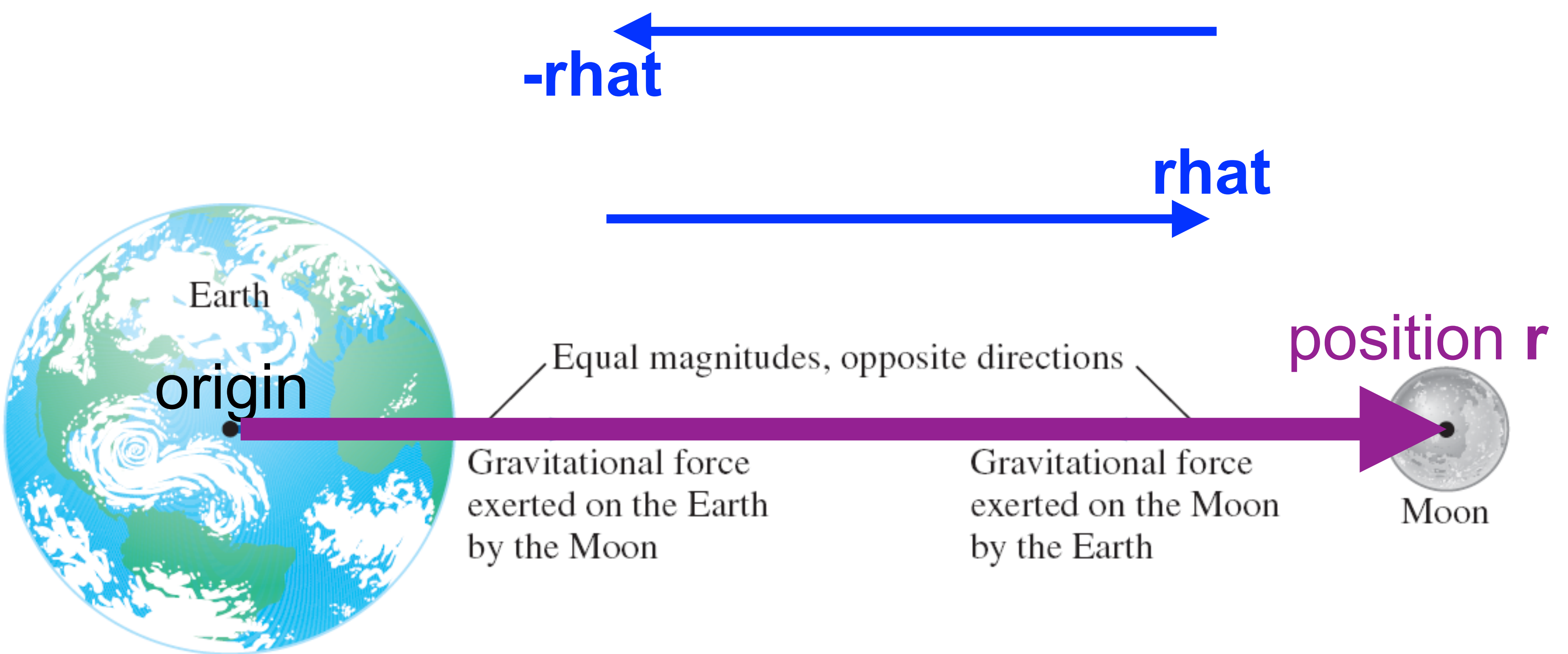


Gravitational force has to be a vector, since forces are vectors. If we place particle 1 (ie Earth) at the origin and object 2 (ie Moon) at position **r**, then the force objects 1 exerts on object 2 is in the -**rhat** direction.

GRAVITY ATTRACTS



Of course, if Earth pulls the moon to the left here, then the moon pulls the Earth to the right with a force equal in magnitude (but thus in opposite direction). That is what Newton's third Law tells us!



An important feature of **Newton's law of universal gravitation** is that it is an "inverse square law". In other words, $|F| \propto 1/r^2$. If the distance between two masses increases by a factor of 5x, the gravitational force between them decreases by a factor of 25x!

But if the mass of one object increases by 5x, the gravitational force it exerts only increases by 5x

$$F = \frac{Gm_1m_2}{r^2}$$

Some people insist on believing that the positions of the planets at the time of your birth can have an influence on your life, a subject sometimes referred to as astrology. Some astrologers even insist that there is a physical basis in astrology because the gravitational pull of the various planets affects a newborn's body.

a) Estimate the magnitude of the gravitational force on a newborn from the planet Jupiter and compare it with the magnitude of the gravitational force due to the pediatrician who happens to be present during the birth.
(For your calculations, assume 4.1 kg for the mass of a newborn, 78 kg for the mass of the pediatrician, 7.8×10^{11} m for the distance to Jupiter, 1.9×10^{27} kg for the mass of Jupiter, and 0.5 m for the distance between the pediatrician and the baby.)

b) Does this (or astrology) make any sense?

Estimate the magnitude of the gravitational force on a newborn from the planet Jupiter and compare it with the magnitude of the gravitational force due to the pediatrician who happens to be present during the birth.
(For your calculations, assume 4.1 kg for the mass of a newborn, 78 kg for the mass of the pediatrician, 7.8×1011 m for the distance to Jupiter, 1.9×1027 kg for the mass of Jupiter, and 0.5 m for the distance between the pediatrician and the baby.)
b) Does this (or astrology) make any sense?

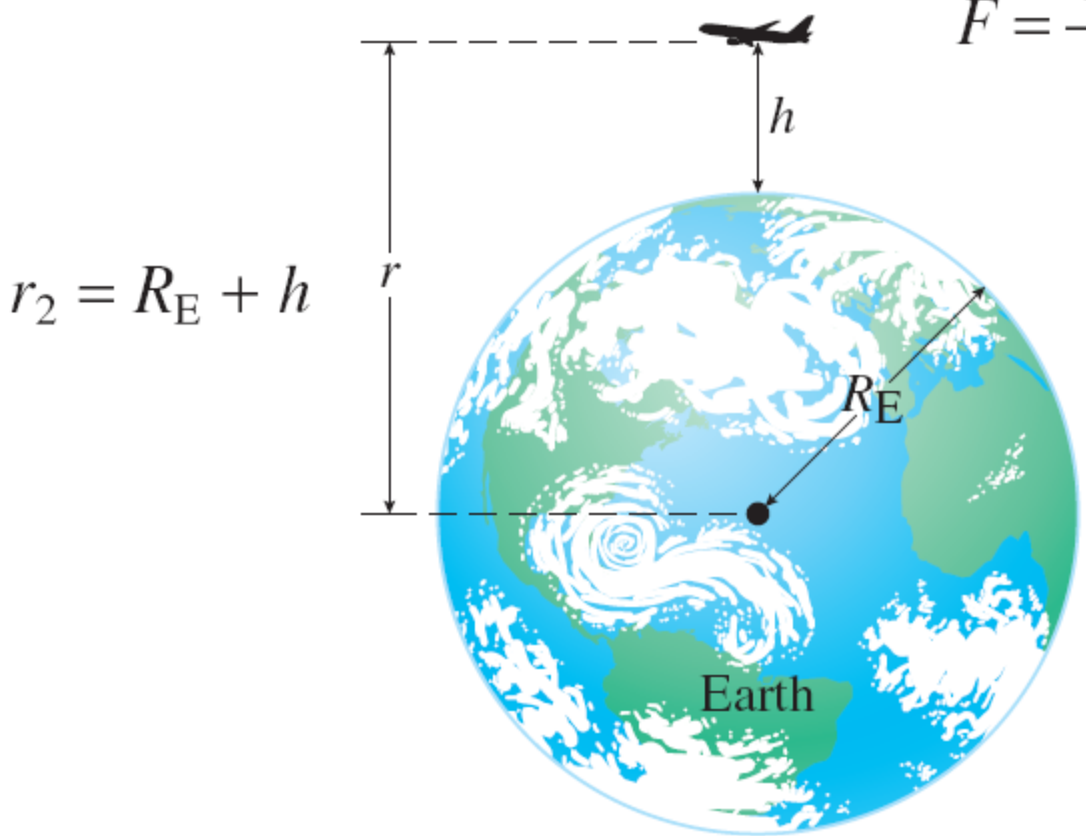
$$F_{jup} = \frac{Gm_j m}{r_j^2} = 8.6x10^{-7} N$$

$$F_{ped} = \frac{Gm_j m}{r_{ped}^2} = 8.6x10^{-8} N$$

So… 10 people in the room would have the same effect! That makes no sense! Astrology is not real. I promise you. Please ignore it. It is fake. It is not science.

When you are in a commercial airliner cruising at an altitude of 6.4 km (21,000 ft), by what percentage has your weight (as well as the weight of the airplane) changed compared with your weight on the ground?

A picture



$$F = \frac{Gm_1m_2}{r^2}$$

$$r_1 = R_E = 6.37 \times 10^6 \text{ m}$$

CAREFUL! It is distance from the **center**, not from the surface, that matters

Solution

$$\frac{W_2}{W_1} = \frac{\dfrac{\cancel{GM_E m}}{r_2^2}}{\dfrac{\cancel{GM_E m}}{r_1^2}} = \frac{r_1^2}{r_2^2} = \frac{R_E{}^2}{(R_E + h)^2}$$

$$= \left(\frac{6.37 \times 10^6 \text{ m}}{6.37 \times 10^6 \text{ m} + 6.4 \times 10^3 \text{ m}}\right)^2 = 0.998$$

Since $0.998 = 1 - 0.002$ and $0.002 = 0.2/100$, your weight decreases by 0.2%.

Gravitational Field Strength

For an object near Earth's surface, the distance between the object and the Earth's center is very nearly equal to the Earth's mean radius.

The weight of an object of mass *m* near Earth's surface is:

$$W = \frac{GM_E m}{R_E^2} = m\left(\frac{GM_E}{R_E^2}\right)$$

$$R_E = 6.37 \times 10^6 \text{ m}$$
$$M_E = 5.97 \times 10^{24} \text{ kg}$$

Gravitational field strength *g*:

$$g = \frac{GM_E}{R_E^2} = \frac{6.674 \times 10^{-11} \text{ N}\cdot\text{m}^2\cdot\text{kg}^{-2} \times (5.97 \times 10^{24} \text{ kg})}{(6.37 \times 10^6 \text{ m})^2} \approx 9.8 \text{ N/kg}$$

Now we see where g comes from!

Gravitational Field

We can define a **gravitational field** produced by a source that is a vector independent of the object it exerts a force on. For example, for the Earth, define **g**:

$$\vec{g} = -\frac{GM_E}{R_E^2}\hat{r}$$

Why the minus sign? Because it is an attractive force. Let's check this as before

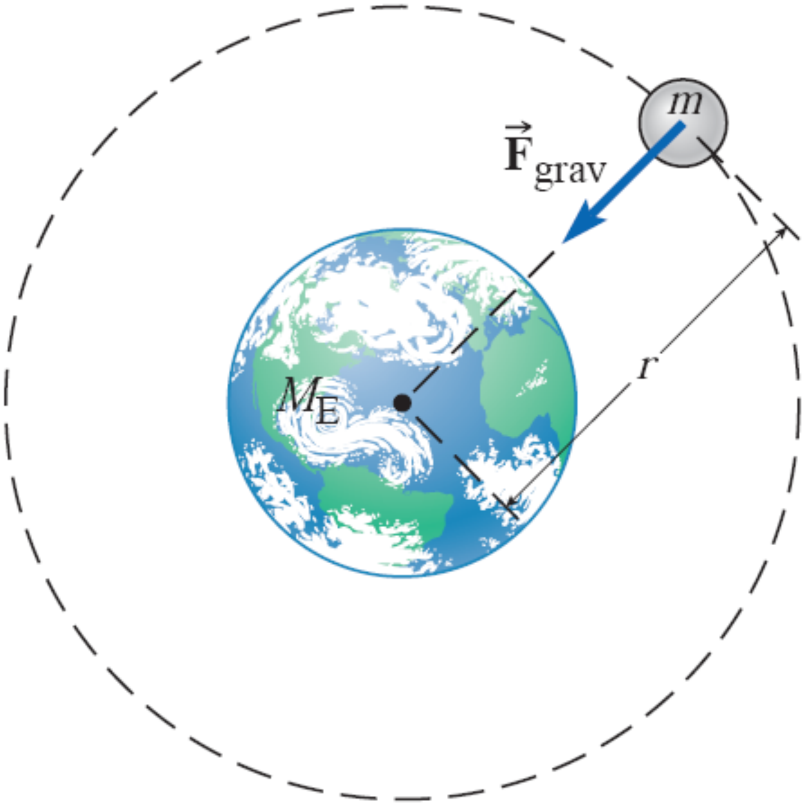
Then the gravitational force on an object with mass m near the Earth can be written as **F** = m**g**. Why is this nice? Because **g** is independent of the object!

Two forms for gravitational force

We have F_{weight}=mg and also $F_{grav}=\dfrac{GM_e m}{r^2}$, so which one should we use? Answer: The full form from Newton's Law of Gravitation (second one) is the fully correct one, but we can write it as $F = \dfrac{GM_e}{(R_E + h)^2}m$. If your height (h) above the surface of the Earth is much smaller than the radius of the earth (R_E) then r=(R_E+h)~R_E and we get $F = m\left(\dfrac{GM_e}{R_E^2}\right) = mg$! So you should check how distances above the surface of the Earth compare to R_E

Application of Radial Acceleration : Circular Orbits

A satellite can orbit Earth in a circular path because of the long-range gravitational force on the satellite due to the Earth.



The magnitude of the gravitational force on the satellite is

$$F = \frac{Gm_1m_2}{r^2}$$

Application of Radial Acceleration : Circular Orbits

Since gravity is the only force acting on the satellite,

$$\sum F_r = G\frac{mM_E}{r^2}$$

where *r* is the distance from the ***center*** of the Earth to the satellite.

Then, from Newton's second law,

$$\sum F_r = ma_r = \frac{mv^2}{r}$$

$$G\frac{\cancel{m}M_E}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$G\frac{\cancel{m}M_E}{r^{\cancel{2}}} = \frac{\cancel{m}v^2}{\cancel{r}}$$

$$v = \sqrt{\frac{GM_E}{r}}$$

Edwin Hubble lived in nearby Wheaton, IL for many years

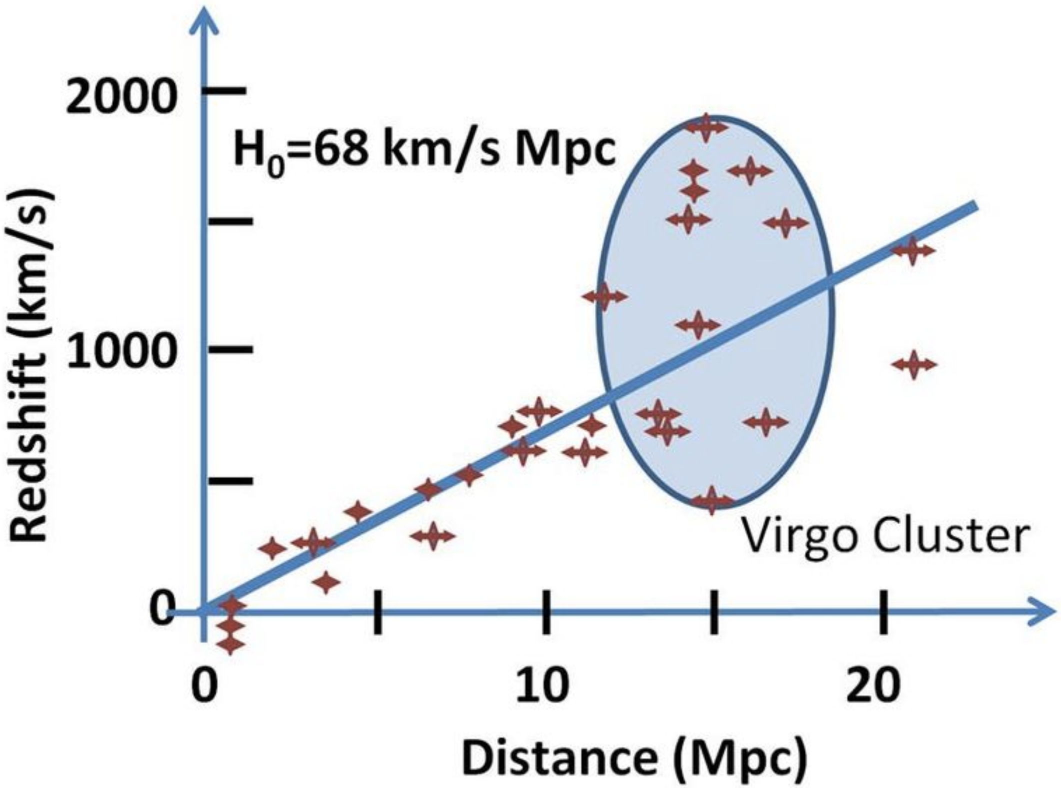
Performed ground-breaking astronomy measurements proving that the Universe is expanding.

NASA's Hubble Space Telescope is named after him

https://upload.wikimedia.org/wikipedia/commons/3/32/Hubble_01.jpg

Pillars of Creation in the Eagle Neubula

https://upload.wikimedia.org/wikipedia/commons/thumb/6/68/Pillars_of_creation_2014_HST_WFC3-UVIS_full-res_denoised.jpg/982px-Pillars_of_creation_2014_HST_WFC3-UVIS_full-res_denoised.jpg



Hubble's Law:
There is a linear relationship between the distance between us and a galaxy and how fast it is moving away from us.

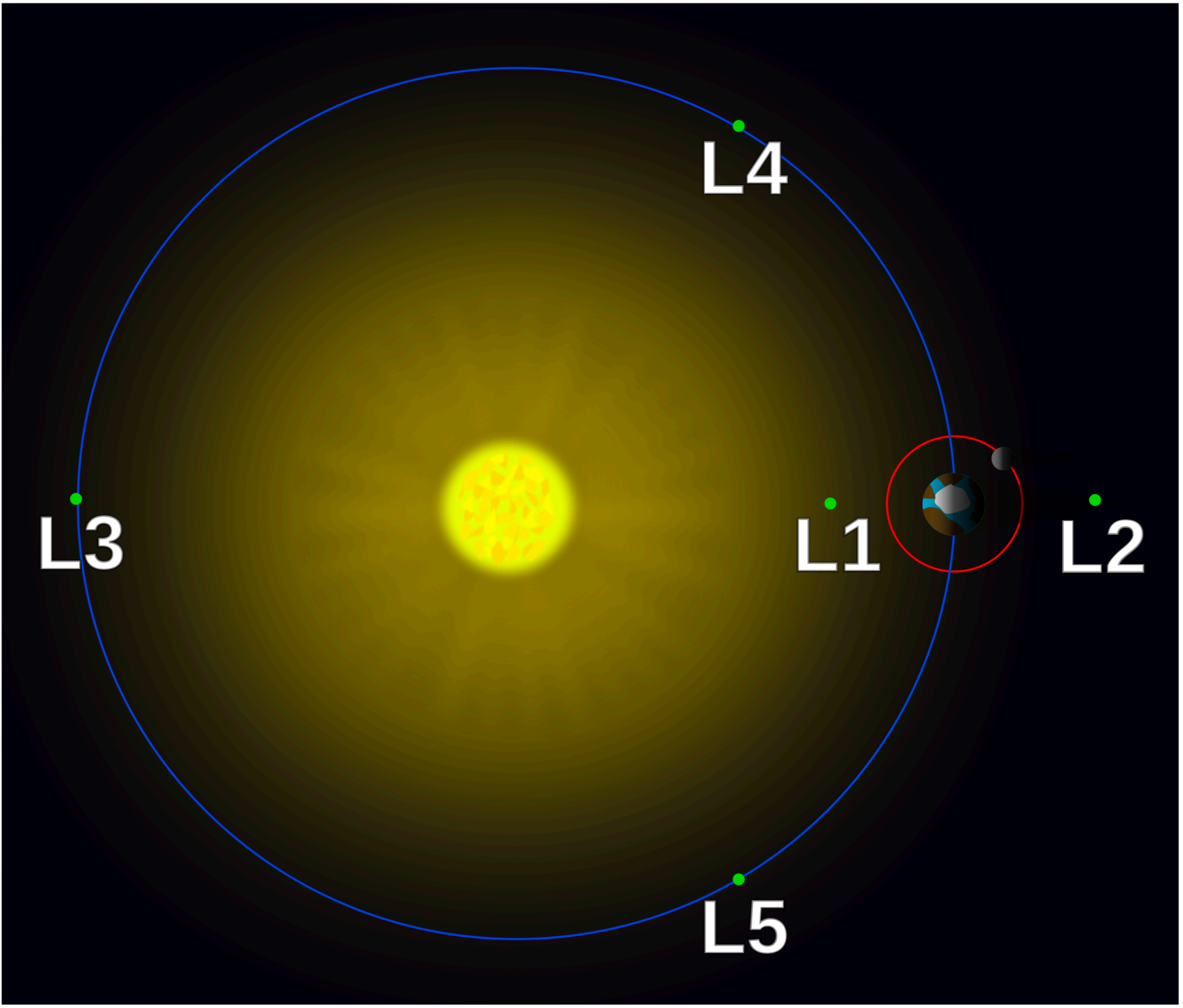
Implies an expanding Universe

https://en.wikipedia.org/wiki/Hubble's_law

The Hubble Space Telescope is … old news!

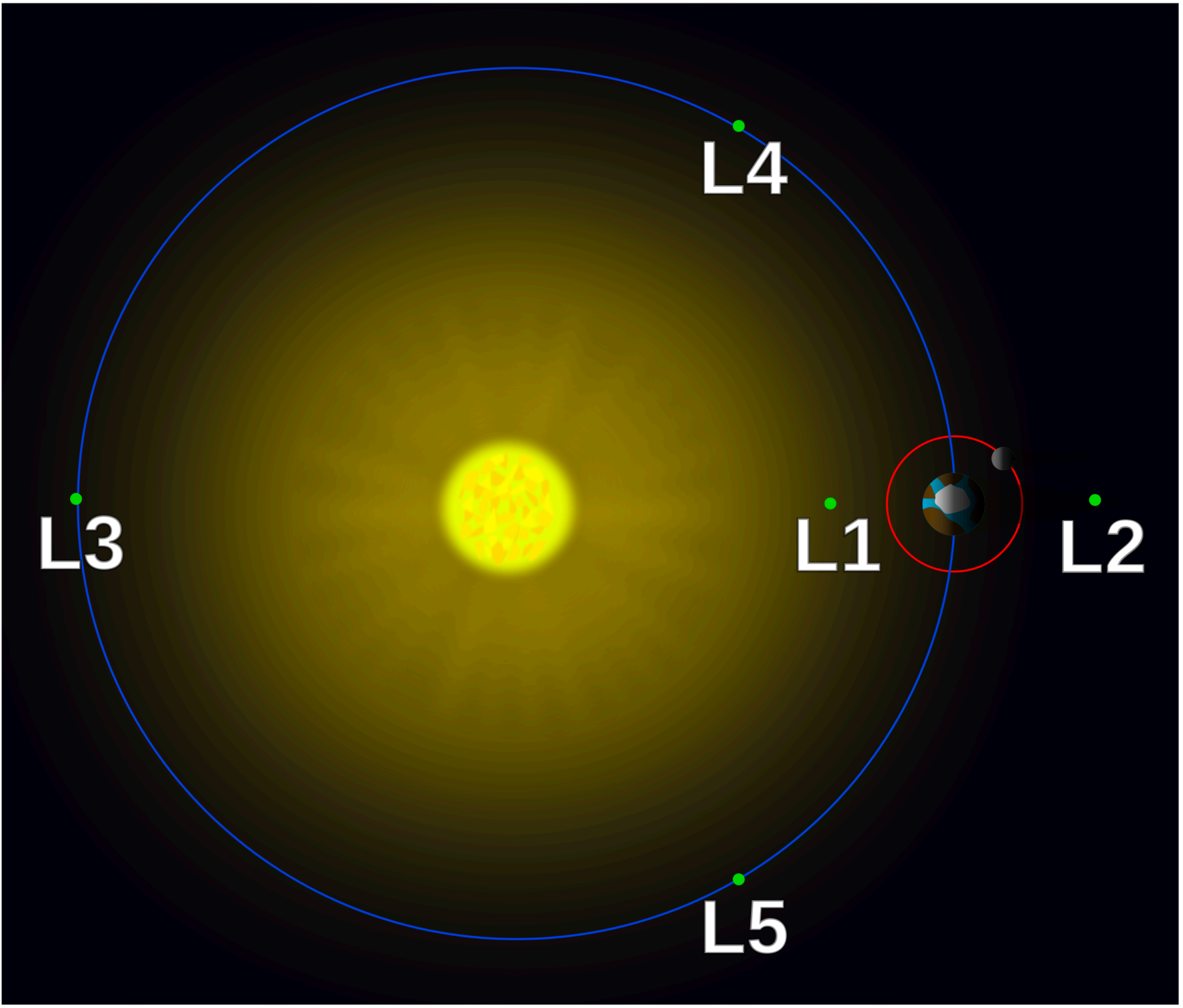
Hello to the era of the James Web Space Telescope

"Cosmic Cliffs" of the Carina Nebula - one of JWST's first photos



JWST orbits at the L2 Lagrange Point. Lagrange Points are spots of equilibrium in the Earth-sun orbit! Here, the combined gravitational force of the Earth and Sun combine to provide the necessary centripetal force for JWST to have the same period as that of the Earth

https://en.wikipedia.org/wiki/Lagrange_point#/media/File:Lagrange_points_simple.svg



Any ideas as to why L2 is the best spot for JWST?

https://en.wikipedia.org/wiki/Lagrange_point#/media/File:Lagrange_points_simple.svg

Application of Radial Acceleration : Geostationary Orbits

Many satellites, such as those used for communications, are placed in a ***geostationary*** (or ***geosynchronous***) orbit—a circular orbit in Earth's equatorial plane whose period is equal to Earth's rotational period.

A satellite in geostationary orbit remains directly above a particular point on the equator.

Why might this be or important?

A 300.0-kg communications satellite is placed in a geostationary orbit 35,800 km above a relay station located in Kenya.

What is the speed of the satellite in orbit?

Solution

$$\sum F_{\mathrm{r}} = \frac{GmM_{\mathrm{E}}}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM_{\mathrm{E}}}{r}}$$

$$r = h + R_{\mathrm{E}} = 3.58 \times 10^7\ \mathrm{m} + 0.637 \times 10^7\ \mathrm{m}$$
$$= 4.217 \times 10^7\ \mathrm{m}$$

$$v = \sqrt{\frac{6.67 \times 10^{-11}\ \mathrm{N{\cdot}m^2/kg^2} \times 5.97 \times 10^{24}\ \mathrm{kg}}{4.217 \times 10^7\ \mathrm{m}}}$$

$$= \sqrt{9.443 \times 10^6\ \mathrm{m^2/s^2}}$$

$$v = 3.07 \times 10^3\ \mathrm{m/s}$$

We found that for an object rotating around a heavy center (such as a satellite around Earth, or stars around the center of a galaxy):

$$v = \sqrt{\frac{GM_E}{r}}$$

In other words, if you have two stars rotating around the center of a galaxy, the one further away (larger r) should have a smaller velocity (v). When scientists look, what do we observe?

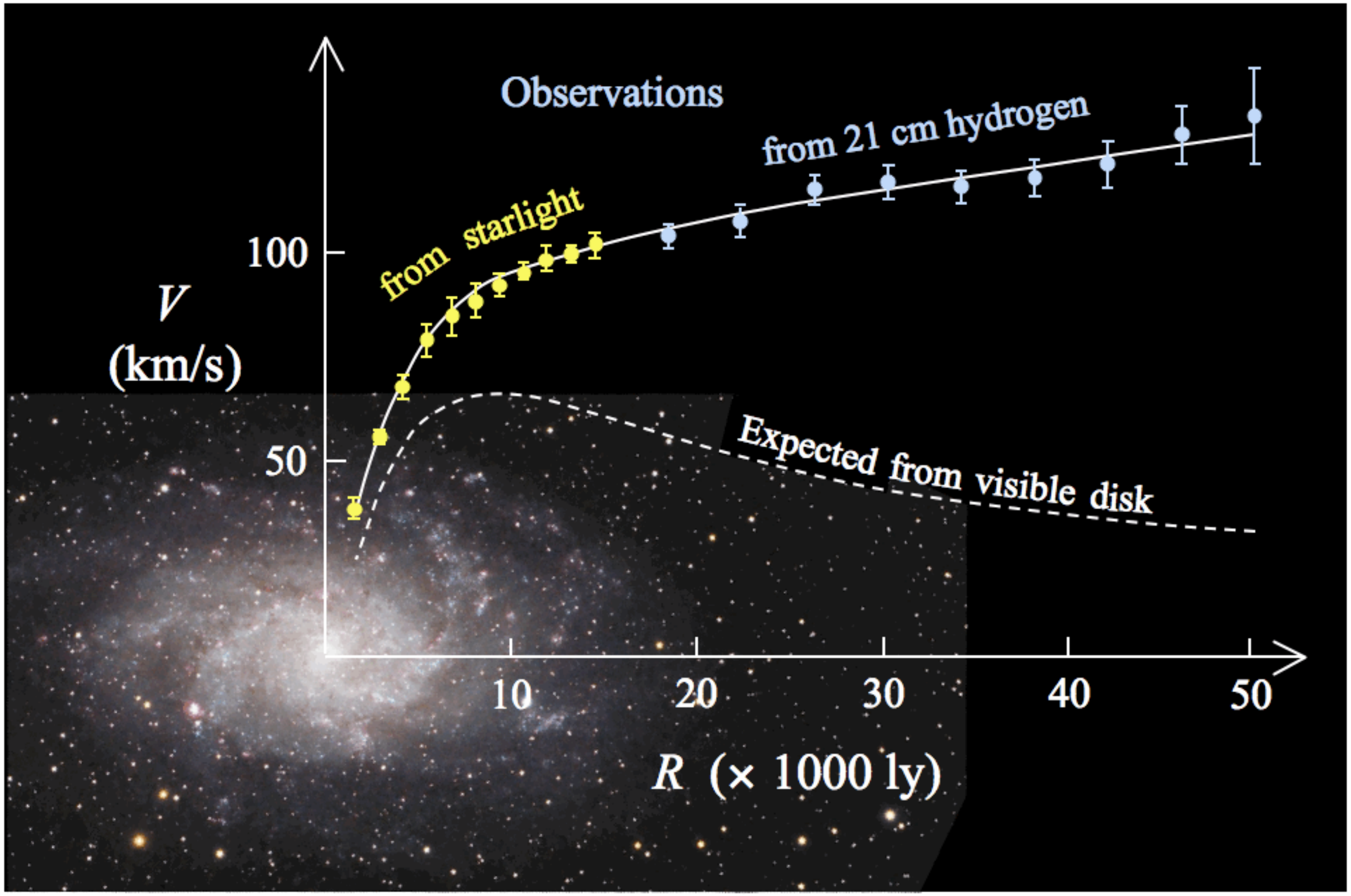
Well, we need to be careful, because we need to know how much mass there is in the galaxy. But we should be able to determine this by how much light we see!

https://upload.wikimedia.org/wikipedia/commons/thumb/c/c5/M101_hires_STScI-PRC2006-10a.jpg/1280px-M101_hires_STScI-PRC2006-10a.jpg

Vera Rubin (1928-2016) attempted to enroll in graduate studies at Princeton in 1948, but they did not admit women (sadly, not until 1975)! We've come a long way… And have a long way to go.

Rubin studied galaxy rotation curves in the 1970s, and found some odd features.

Let's look closely and careful at what is being shown here

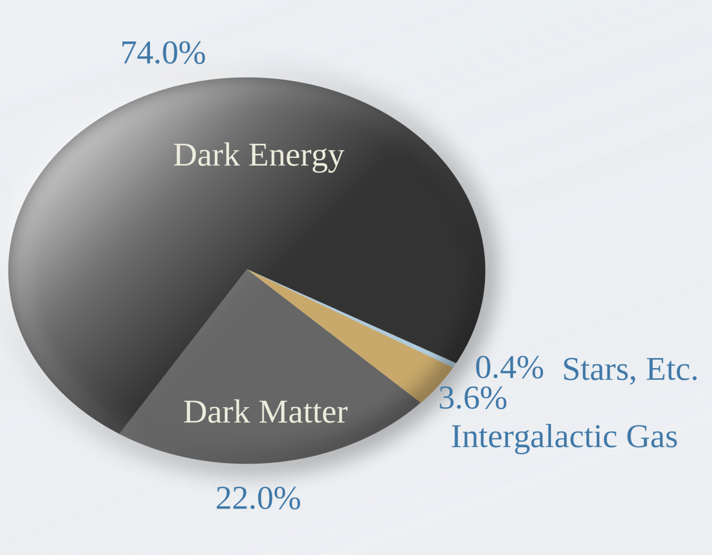


Clear disagreement between prediction and observation! Three possibilities:

1) Newton's laws are wrong
2) Our understanding of gravity is wrong
3) As you go further out from the center of a galaxy, there is more matter that has gravitational pull that we cannot otherwise see in any other way

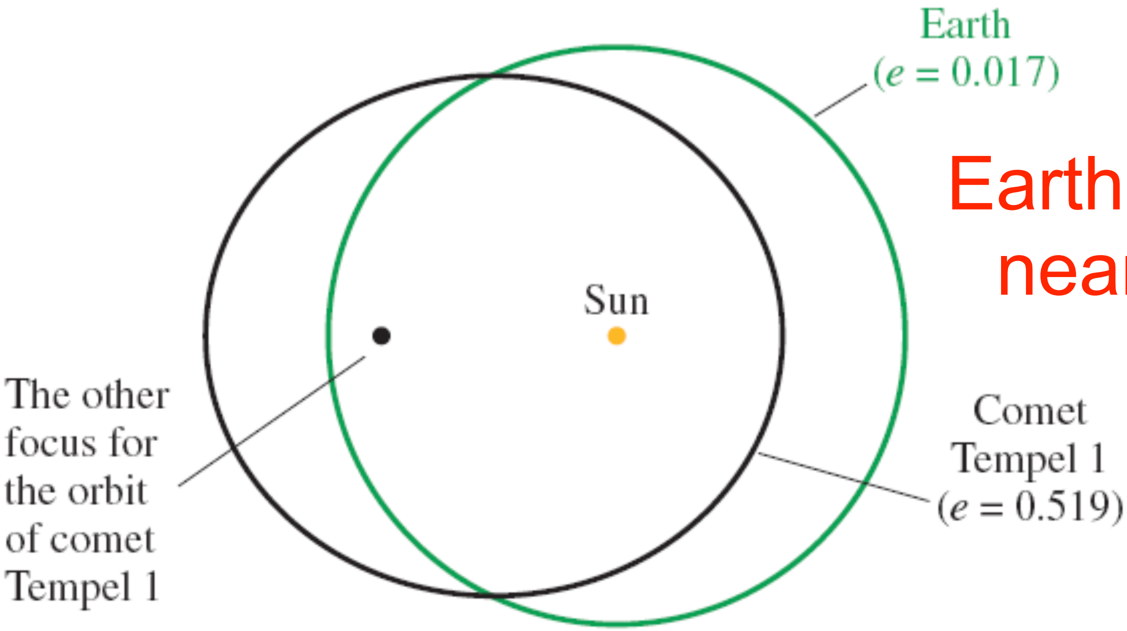
1) Newton's laws are wrong (physicists don't typically like this, though we test for it, and see no reason otherwise why Newton was wrong!)
2) Our understanding of gravity is wrong (a possibility, but other tests of gravity show good agreement between observation and prediction!)
3) As you go further out from the center of a galaxy, there is more matter that has gravitational pull that we cannot otherwise see in any other way (in other words, there is a lot of matter out there that we cannot see, that does not interact with light, and does not emit light, but that has gravitational influence. We call this 'dark matter'. We see strong evidence for dark matter in other ways, too! Finding dark matter has been a holy grail of physics for several decades)

Vera Rubin (via Wikipedia): "If I could have my pick, I would like to learn that Newton's laws must be modified in order to correctly describe gravitational interactions at large distances. That's more appealing than a universe filled with a new kind of sub-nuclear particle."



Kepler's Laws of Planetary Motion

1. The planets travel in elliptical orbits with the Sun at one focus of the ellipse.



Earth-sun orbit is nearly circular

Closest position of planet to sun = **perihelion**
Farthest position of planet to sun = **aphelion**

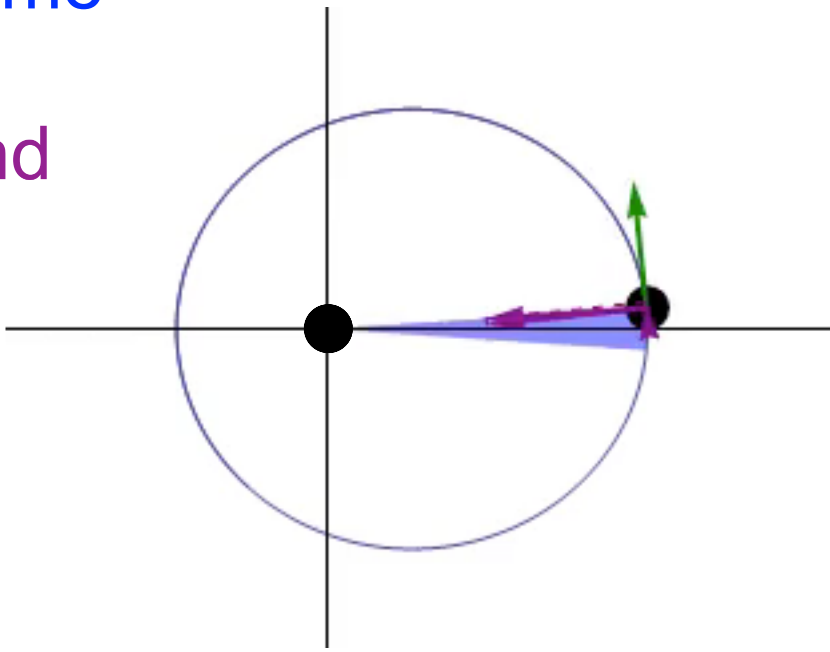
Kepler’s Laws of Planetary Motion

2. A line drawn from a planet to the Sun sweeps out equal areas in equal time intervals.

Blue = Area swept out per unit time

Green arrow = velocity

Purple arrow = acceleration and its components



Speed is highest when planet is nearest the sun!

https://en.wikipedia.org/wiki/Kepler%27s_laws_of_planetary_motion

Kepler's Laws of Planetary Motion

3. The square of the orbital period is proportional to the cube of the average distance from the planet to the Sun. In other words, T^2/a^3 = constant! (a = average distance, which is R for a circular orbit)

Derivation of Third Law for a circular orbit

$$\sum F_r = \frac{GmM_{\text{Sun}}}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM_{\text{Sun}}}{r}}$$

$$v = \sqrt{\frac{GM_{\text{Sun}}}{r}} = \frac{2\pi r}{T}$$

Velocity = distance/time

$$T = 2\pi\sqrt{\frac{r^3}{GM_{\text{Sun}}}}$$

Let's check this one

$$T^2 = \frac{4\pi^2}{GM_{\text{Sun}}} r^3 = \text{constant} \times r^3$$

Derivation of Third Law for a circular orbit

$$T^2 = \frac{4\pi^2}{GM_{\text{Sun}}} r^3 = \text{constant} \times r^3$$

Kepler's third law: the square of the period of a planet is directly proportional to the cube of the average orbital radius.

Application : Apparent Weightlessness of Orbiting Astronauts

Comparing an astronaut's weight in orbit with her weight on Earth's surface,

$$\frac{W_{\text{orbit}}}{W_{\text{surface}}} = \frac{\dfrac{GM_{\text{E}}m}{(R_{\text{E}}+h)^2}}{\dfrac{GM_{\text{E}}m}{R_{\text{E}}^2}} = \frac{R_{\text{E}}^2}{(R_{\text{E}}+h)^2} = \frac{(6400\ \text{km})^2}{(7000\ \text{km})^2} = 0.84$$

So, why does the astronaut *seem* to be weightless?

Application : Apparent Weightlessness of Orbiting Astronauts

An astronaut feels weightless when.

$$\vec{\mathbf{a}} = \vec{\mathbf{g}},$$

where $\vec{\mathbf{g}}$ is the *local* gravitational field. Compare to this our elevator. What happens if you are in an elevator and the cable snaps and you try and measure your weight on a scale? How does this compare to the astronaut?

Gravity elsewhere

https://imgur.com/gallery/l2xdmlf

Group work time!

https://forms.gle/o7n18xpA1MERaZnd8

At what altitude about the Earth's surface would your weight be half of what it is at the Earth's surface?

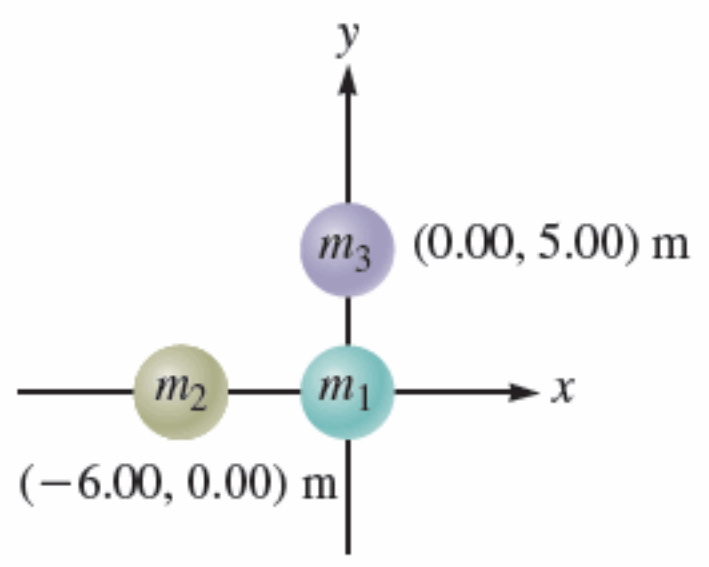
a) Find the Sun's gravitational force on the Earth when the Earth is at its perihelion (earth-sun distance = 1.48 x 10^11 m), and aphelion (earth-sun distance = 1.52 x 10^11 m). Mass of Earth is 5.97 x 10^{24} kg, mass of sun is 1.99 x 10^{30} kg

b) What is the fractional difference between the magnitude of the gravitational force at those two points?

Students often incorrectly assert that the Earth’s seasons are caused by its elliptical orbit around the Sun. Why is that assertion incorrect? Any ideas as to what does cause the seasons, in that case?

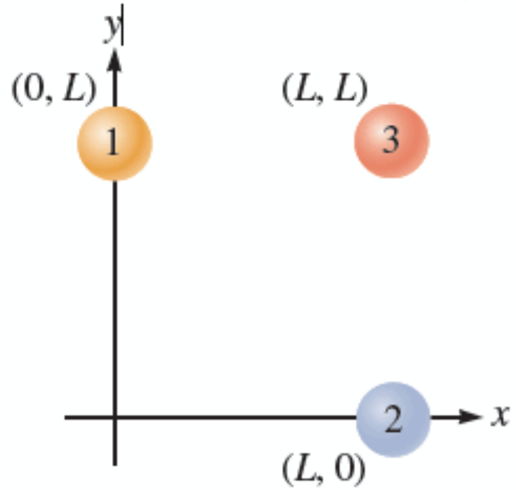
Which is greater in magnitude? The gravitational force of the Sun on an asteroid or the gravitational force of an asteroid on the Sun? Clearly explain your reasoning without any formulas

Three spheres are arranged in the xy plane as in the figure. The first sphere, of mass m1 = 12.5 kg, is located at the origin. The second sphere, of mass m2 = 4.50 kg, is located at (-6.00, 0.00) m, and the third sphere, of mass m3 = 8.00 kg, is located at (0.00, 5.00) m. Assuming an isolated system, what is the net gravitational force on the sphere located at the origin?



Three particles, each with mass m, are located at coordinates (0,L), (L,0) and (L,L) as in the figure.

a) What are the magnitude and direction of the gravitational field at the origin due to the three particles?
b) What would the gravitational force be on an object of mass m located at the origin?



Maria, with mass $m = 48.0$ kg, is in the second row of seats, a distance of 3.00 m from Prof. Karen, who has a mass $M = 55.0$ kg. What is the magnitude of the gravitational force between Maria and Prof. Karen?

When first detected, near-Earth asteroid 2011 MD was at its closest approach of only 12,000 km away from the Earth's surface. What was the asteroid's acceleration due to the Earth's gravity at this point in its trajectory?

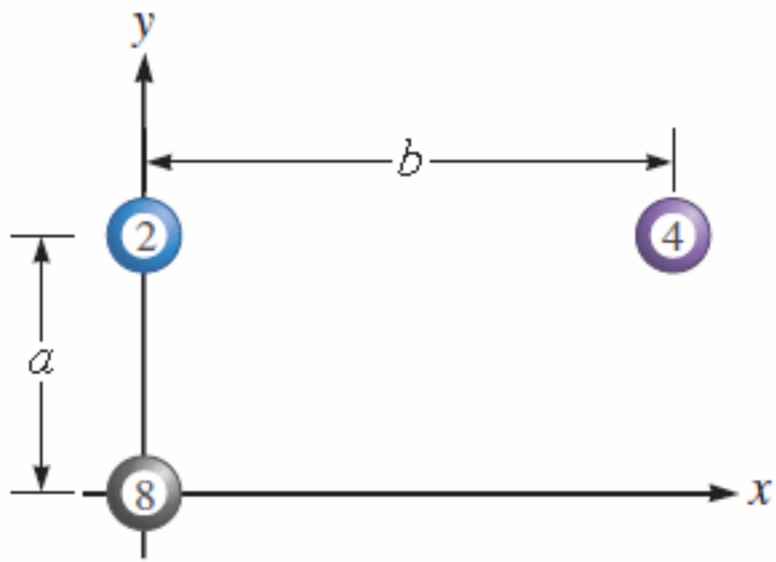
The International Space Station (ISS) experiences an acceleration due to the Earth’s gravity of 8.83 m/s^2. What is the orbital period of the ISS?

Two black holes separated by a distance of 10.0 AU (1 AU = 1.50 x 10^11 m) attract one another with a gravitational force of 8.90 x 10^25 N. The combined mass of the two black holes is 4.00 x 10^30 kg. What is the mass of each black hole?

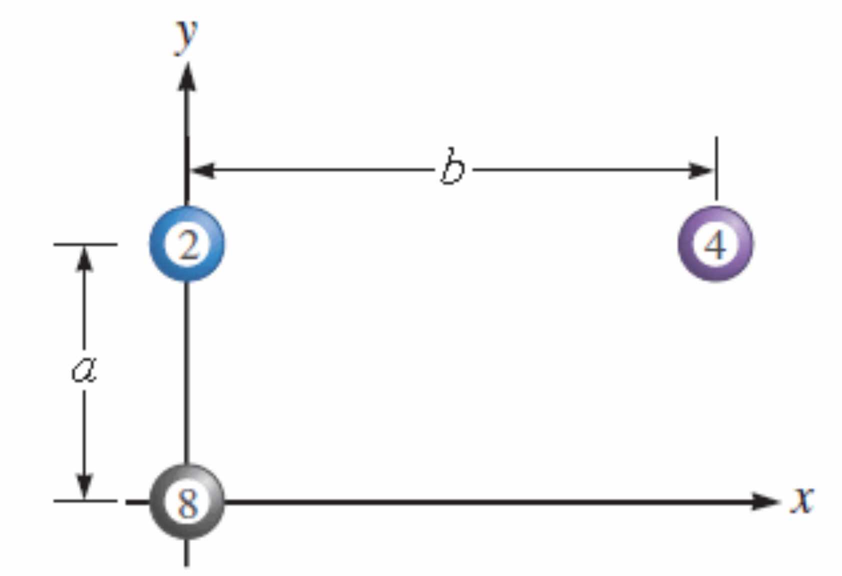
Many science-fiction spacecraft are shown with cylindrical modules that rotate to provide the crew with artificial gravity - a centripetal acceleration that is comparable to the gravitational acceleration on the Earth, 9.81 m/s^2. If one such module is 500 m in diameter, what is the speed of rotation that would provide an artificial gravity equal to that on the Earth's surface?

Suppose a planet with mass 2.44×10^{25} kg is orbiting a star with a mass of 3.65×10^{31} kg and the mean distance between the planet and the star is 1.12×10^{12} m. Determine the speed of the planet when it is at the mean distance from the star.

Three billiard balls, the two-ball, the four-ball, and the eight-ball, are arranged on a pool table as below. Given the coordinate system shown and that the mass of each ball is 0.150 kg, determine the gravitational force on the eight-ball due to the other two balls if a = 1m and b = 2m.

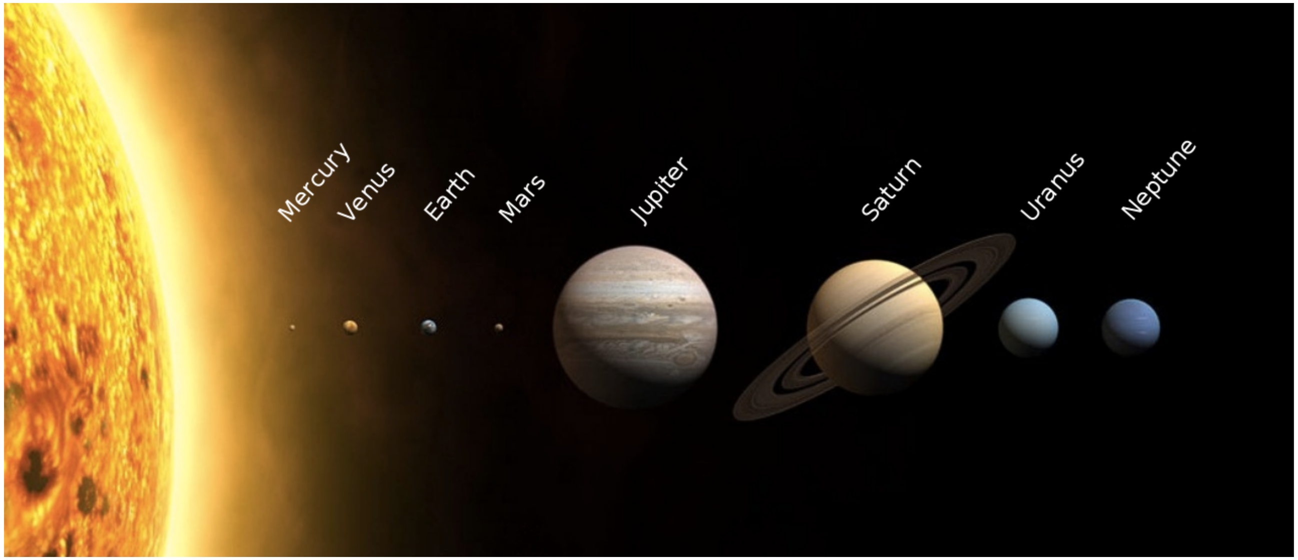


Three billiard balls, the two-ball, the four-ball, and the eight-ball, are arranged on a pool table as below. Given the coordinate system shown and that the mass of each ball is 0.150 kg, determine the gravitational field at x = 2m, y = 0m if a = 1m and b = 2m.



Assume that the Earth's orbit is circular (not quite but close to correct) and that the distance between the sun and the Earth is 93 million miles.

If the orbit of Mars is also circular (again, close to correct) and the distance between Mars and the sun is 142 million miles, how long does Mars take to make one full revolution around the sun?



The Hubble Space Telescope orbits Earth 613 km above Earth's surface. What is the period of the telescope's orbit?

The Hubble Space Telescope is in a circular orbit 613 km above Earth's surface. The average radius of the Earth is 6.37×10^3 km and the mass of Earth is 5.97×10^{24} kg.

What is the speed of the telescope in its orbit?

The distance between mercury and the sun is $6.9x10^7$ km. Assume mercury has a circular orbit. What is its speed around the sun? The mass of the sun is $2.0x10^{30}$ kg

A satellite revolves about Earth with an orbital radius of r_1 and speed v_1.

If an identical satellite were set into circular orbit with the same speed about a planet of mass three times that of Earth, what would its orbital radius be?